Theoretische Physik IV: Statistische Mechanik und Thermodynamik

Problem Set No. 1

Due on: Friday, 25.4.08 in the practice groups

Exercise 1.1 *(Phase Space of a Free Particle)* **(10 points)**

A free particle with mass m is located inside a (one-dimensional) box of length L with infinitely high walls.

(a) Calculate the volume of the *classical* phase space with energies in the interval $[E - \Delta, E + \Delta]$. What is the result for $\Delta \ll 1$? (3 points)

(b) Calculate the volume of the *classical* phase space with energies smaller or equal than E. (2 points)

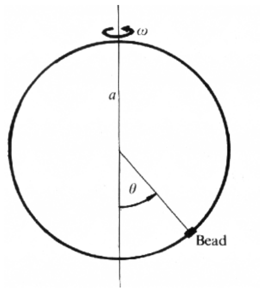
(c) What is the number of states with energies smaller than E for the corresponding *quantum mechanical* system? Compare your result with the classical one for large E. (5 points)

Exercise 1.2 *(Phase Space Trajectories)* **(10 points)**

Consider a system with kinetic energy T and potential energy V given by:

$$T = \frac{1}{2} m a^2 (\dot{\theta}^2 + \omega^2 \sin^2 \theta) \qquad V = -mga\cos\theta$$

This system describes a bead sliding on a circular wire with radius a, which is constrained to rotate about a vertical diameter with constant angular velocity ω (see figure).



(a) Calculate the equation of motion of this system by means of the corresponding Lagrange equation: (2 points)

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = -\frac{\partial V}{\partial \theta}$$

(b) What are the phase space trajectories $f(\theta, \dot{\theta}) = C(\equiv \textit{const.})$ for this system? (3 points)

(c) Determine the equilibrium points of the system, i.e. $\ddot{\theta} = \dot{\theta} = 0$ (1 point)

(d) We now want to analyze the phase space trajectories near the point $(\theta, \dot{\theta}) = (0, 0)$: Perform a series expansion of the equation $f(\theta, \dot{\theta}) = C$ obtained in (b) up to the second order in $\dot{\theta}$ and θ. How do the trajectories look like for $a\omega^2/g < 1$ and $a\omega^2/g > 1$? Draw a family of trajectories in phase space for both cases. (4 points)

Exercise 1.3 *(Density Operator)* **(10 points)**

Let $|\psi_i\rangle$ be normalized Hilbert space states and $p_i \in [0,1]$ with $\sum_i p_i = 1$. Then the density operator $\hat{\rho}$ is defined by

$$\hat{\rho} = \sum_i p_i |\psi_i\rangle \langle\psi_i|$$

Consider now a two-dimensional hilbert space with orthonormal basis $\{|1\rangle, |2\rangle\}$ and density operator

$$\hat{\rho} = \alpha |1\rangle \langle 1| + \frac{1}{2} |x\rangle \langle x| \qquad \text{mit } |x\rangle = \frac{|1\rangle + |2\rangle}{\sqrt{2}}$$

(a) Determine α such that $\hat{\rho}$ indeed represents a density operator. (3 points)

(b) Determine the matrix representation of the density operator in the basis $\{|x\rangle, |y\rangle\}$ with $|x\rangle = \frac{|1\rangle+|2\rangle}{\sqrt{2}}$ and $|y\rangle = \frac{|1\rangle-|2\rangle}{\sqrt{2}}$? (4 points)

(c) The system now passes a device which only lets particles in state $|1\rangle$ through (e.g. a Stern-Gerlach-device). What is the density operator of the system after passing the device? (3 points)